Pergamon

PII: S0021-8928(97)00054-3

J. Appl. Maths Mechs, Vol. 61, No. 3, pp. 427–432, 1997

0021–8928/97 $24.00+0.00

RECONSTRUCTION OF THE SET OF CONTROLS BY MEASUREMENTS OF THE STATES OF AN EVOLUTION SYSTEM†

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(*Received* 1 *August* 1995)

An inverse problem of dynamics is considered, namely, to reconstruct a set of a priori unknown controls (perturbations) applied to a non-linear dynamical system and generating an observable motion of the system. The information for the reconstruction is obtained by approximate measurements of the changing phase states of the system; the reconstruction is carried out in real time in the Hausdorff metric. This problem is known to be ill posed. It is proposed to solve it using constructive, physically realizable, regularizing finite-staged position algorithms for the stable dynamical approximation of the unknown set.

This paper continues the researches described in [1–5]; in its basic ideas it relies on previous results [6–9]. Similar approaches to determining the parameters of dynamical systems, based on guaranteed estimation methods, were considered, *inter alia*, in [10–13].

1. STATEMENT OF THE PROBLEM

Consider a controlled dynamical system which, over a time interval $T = [t_0, \vartheta]$ $(-\infty < t_0 < \vartheta < +\infty)$, is described by the non-linear equation

$$\dot{y}(t) + A(t)y(t) = B(t, y(t))u(t) + f(t), \quad y(t_0) = y_0 \tag{1.1}$$

The system is driven by a priori unknown controls $u = u(t)$. For each admissible control $u \in \Sigma$ the system has a well-defined motion, $y = y(u) = y(t; u)$. It is required, by observing some motion of the system and by making approximate measurements of the changing states $y(t)$, to determine in real time all the controls $u \in \Sigma$ that generate that motion. Throughout this paper, unless otherwise stated, $t \in T$.

Let V be a separable reflective Banach space, embedded continuously and densely in a Hilbert space H. Identifying H with its adjoint H^*, we obtain a triple of continuously and densely embedded spaces $V \subset H \subset V^*$ (to fix our ideas, we assume that $C_V||\cdot||_V \geqslant C_H||\cdot||_H \geqslant ||\cdot||_{V^*}$). Let p and p_0 be given numbers such that $1 < p \leqslant p_0 < \infty$, $2 \leqslant p_0$. Define $X = L^p(T; V) \cap L^{p_0}(T; H)$; then $X^* = L^q(T; V^*) + L^{q_0}(T; H)$, where $1/p + 1/q = 1$, $1/p_0 + 1/q_0 = 1$. Let $W = \{y \in X : \dot{y} \in X^*\}$, where $\dot{y}$ is the derivative of y in the sense of the distribution space $||y||_W = ||y||_X + ||\dot{y}||_X$. The definitions of function spaces that we are using, as well as the main properties of such spaces, may be found, for example, in [14, 15]. In particular, X and W are reflective Banach spaces and W is continuously embedded in $C(T; H)$.

For any $y, z, \in W$, we have the following formula of integration by parts

$$\langle y(t), z(t)\rangle - \langle y(s), z(s)\rangle = \int_s^t \left(\langle \dot{y}(\tau), z(\tau)\rangle + \langle \dot{z}(\tau), y(\tau)\rangle\right)d\tau$$

If $y = x$, this formula becomes

$$\|y(t)\|_H^2 - \|y(s)\|_H^2 = 2\int_s^t \langle \dot{y}(\tau), y(\tau)\rangle\, d\tau$$

where $\langle\cdot,\cdot\rangle$ is the canonical duality between V^* and V; its restriction to H is the scalar product in H, $s \in T$.

For each element $y, \in W$, the Newton–Leibniz formula holds in the space V^*

†*Prikl. Mat. Mekh.* Vol. 61, No. 3, pp. 440–446, 1997.

$$y(t) = y(s) + \int_s^t \dot{y}(\tau)\, d\tau$$

Each element $y \in W$ is strongly differentiable over T as a mapping $T \to V^*$, and the corresponding strong derivative is identical almost everywhere in T with the derivative $\dot{y}$.

Let $\{A(t) : t \in T\}$ be a family of radially continuous monotone operators from V to V^* such that $\langle A(t)v, w\rangle$ is a continuous function for any fixed $v, w \in W$, $\langle A(t)y(t), z(t)\rangle$ is a measurable functional of t for any fixed $y, z \in X$, and constants $C_1 > 0$, $C_2 \in R$, $C_3 > 0$ exist such that, for any $t \in T$, $v \in V$

$$\langle A(t)v, v\rangle \geqslant C_1 \|v\|_V^p - C_2, \quad \|A(t)v\|_{V^*} \leqslant C_3\left(\|v\|_V^{p-1} + 1\right)$$

Let $\{B(t, z) : t \in T, z \in H\}$ be a family of continuous linear operators acting from a finite-dimensional Hilbert space into H, in such a way that the mapping $T \times H \ni (t, z) \to B(t, z) \in L(U; H)$ is continuous and for any $t \in T$, $z \in H$

$$\|B(t, x) - B(t, z)\|_{L(U;H)} \leqslant C_4 \|x - z\|_H$$

$$\|B(t, z)\|_{L(U;H)} \leqslant C_5\left(\|z\|_H + 1\right)$$

where C_4 and C_5 are certain non-negative constants, and $L(U; H)$ is the Banach space of continuous linear operators from U to H with natural norm.

Let $y_0 \in H$, $f \in X^* \cap C(T; V^*)$, let P be a convex bounded closed set of elements of U, and let Σ be the set of all measurable mappings $T \to P$ (this set is convex, bounded and closed in the space $L^r(T; U)$, $1 \leqslant r < \infty$). The set P is the set of instantaneous bounds for the admissible controls in Σ.

Under the assumptions just outlined concerning the parameters, the Cauchy problem (1.1) has a unique solution in the space W. Moreover, the mapping

$$H \times X^* \times L^r(T; U) \ni (y_0, f, u) \to y = y(y_0, f, u) \in C(T; H)$$

is continuous [14, Ch. 6, Section 1]. The solution $y = y(u)$ will also be treated as the motion of the dynamical system (1.1) over a time T in the phase space H from its initial state y_0 when driven by the control $u \in \Sigma$. Let $Y = \{y(u) : u \in \Sigma\}$ be the set of all possible motions of system (1.1) from the initial state y_0 (the set Y is bounded in W).

For each motion $y \in Y$, we introduce the set of all controls

$$\Sigma(y) = \{u \in \Sigma : y(u) = y\}$$

which generate that motion. This set is non-empty and will generally contain more than one element; it is always convex, bounded and closed in $L^r(T; U)$. At each particular time t one can measure the state $y(t)$ of the system; the measurement result $\xi(t)$ is related to the state $y(t)$ by the equation

$$\xi(t) = y(t) + \eta(t), \quad \|\eta(t)\|_V \leqslant h \tag{1.2}$$

The problem is to construct an algorithm which, based on the evolution of the process (in real time) and the results (1.2) of measurements of the changing phase states of an observable motion $y \in Y$ of the dynamical system, will reconstruct the set $\Sigma(y)$ approximately (in the sense of the Hausdorff metric). This should be done in such a way that the reconstruction is more accurate whenever the input data are more accurate. The equation of motion and the set P are assumed known.

Let Ξ be the set of all mappings $T \to V^*$, and let S_h be a closed sphere in V, of radius $h > 0$, with centre at zero, and

$$\Xi_h(y) = \{\xi \in \Xi : \xi(t) = y(t) + \eta(t),\ \eta(t) \in S_h\}, \quad y \in Y$$

Consider the set $\operatorname{Conv}(\Sigma)$ of all non-empty convex bounded closed subsets of $\Sigma \subset L^r(T; U)$; the Hausdorff metric on this set is defined as

$$\rho(\Sigma_1, \Sigma_2) = \max\left\{\sup_{u_1 \in \Sigma_1} \inf_{u_2 \in \Sigma_2} \|u_1 - u_2\|_{L^r(T;U)}, \sup_{u_2 \in \Sigma_2} \inf_{u_1 \in \Sigma_1} \|u_1 - u_2\|_{L^r(T;U)}\right\}$$

We say that a family $(D_h)_{h>0}$ of operators D_h: $\Xi \to \mathrm{Conv}(\Sigma)$ is ρ-regularizing at a point $y \in Y$ if

$$\sup\{\rho(D_h\xi, \Sigma(y)) : \xi \in \Xi_h(y)\} \to 0, \quad h \to 0$$

A solution of the problem will be sought in the class of finite-staged dynamical ρ-regularizing position algorithms (FDPAs). By an FDPA we mean a triple

$$D = \left((\tau_i)_{i=1}^{m}; (Z_i)_{i=0}^{m-1}; (G_i)_{i=0}^{m-1}\right) \tag{1.3}$$

where m is a natural number, $(\tau_i)_{i=0}^{m}$ is a partition of the interval T by points τ_i ($t_0 = \tau_0 < \tau_1 < \ldots < \tau_m = \vartheta$), Z_i is a mapping $T \times T \times V^* \times V^* \times \mathrm{Conv}(\Sigma[\tau_i, \tau_{i+1})) \to V^*$, G_i is a mapping $T \times V^* \times V^* \to \mathrm{Conv}(\Sigma[\tau, \tau_{i+1})$, and $\Sigma[\tau_i, \tau_{i+1})$ is the set of all non-empty convex bounded closed subsets of $\Sigma[\tau_i, \tau_{i+1}) \subset L^r([\tau_i, \tau_{i+1}); U)$. The performance of such algorithms in time and their dynamical realizations have been described in detail in [1–5, 7, 8] (cf. the formation of the controlled process below). In a physical sense, the mappings (Z_i) form the motion of an auxiliary model controlled system, and the mappings (G_i), in a positional way, generate the control impulses for the model system [6].

Given an FDPA (1.3) and a function $\xi \in \Xi$, the pair of elements

$$(z(\cdot), g(\cdot)) = (z(\cdot \mid D, \xi), g(\cdot \mid D, \xi)) \in (T \to V^*) \times \mathrm{Conv}(\Sigma)$$

formed according to the rule

$$z(t_0) = \xi(t_0), \quad z(t) = Z_i(t, \tau_i, z(\tau_i), \xi(\tau_i), g_i(\cdot)), \quad \tau_i < t \leq \tau_{i+1}, \quad i = 0, \ldots, m-1$$
$$g(\cdot)|_{[\tau_i, \tau_{i+1})} = g_i(\cdot) = G_i(\tau_i, z(\tau_i), \xi(\tau_i)), \quad i = 0, \ldots, m-1$$

will be called a controlled process for the FDPA (1.3) and measurement $\xi \in \Xi$. Corresponding to each FDPA (1.3) we have an operator $D : \Xi \to \mathrm{Conv}(\Sigma)$, which we will denote, for simplicity by the same symbol; its action is described by the rule $D\xi = g(\cdot \mid D, \xi)$. This operator will be identified with the FDPA (1.3) itself. Obviously, the operator has an unpredictability property: the condition $\xi_1(\tau) = \xi_2(\tau)$, $t_0 \leq \tau \leq t$, $t \in T$ implies that the restrictions to the interval $[t_0, t]$ of the images $D\xi_1$ and $D\xi_2$ coincide. Note that algorithms for reconstructing controls that have this property are important in practice, e.g. in cases where one is concerned with a single reconstruction, when the computations cannot be repeated, or when the results of the reconstruction are being used in feedback systems.

Thus, to solve the reconstruction problem, it will suffice to construct a ρ-regularizing family of FDPAs. We shall outline some possible constructions of this sort below.

2. CONSTRUCTION OF A REGULARIZING FAMILY OF ALGORITHMS

We will first introduce a few auxiliary notions and notations. For fixed $y \in Y$, we define a multi-valued mapping

$$Q(\cdot \mid y) : T \ni t \to Q(t \mid y) \in \mathrm{Conv}(P)$$

by the rule

$$Q(t \mid y) = \{w \in P : \dot{y}(t) + A(t)y(t) = B(t, y(t))w + f(t)\}$$

if y is differentiable at t and this set is non-empty; otherwise, $Q(t \mid y) = P$. Clearly, $\Sigma(y) = \mathrm{sel}\, Q(\cdot \mid y)$, where $\mathrm{sel}Q(\cdot \mid y)$ is the set of all measurable selectors of the multivalued mapping $Q(\cdot \mid y)$, $Q(\cdot \mid y) \in M(T; P)$, where $M(T; P)$ is the space of all measurable, bounded (hence also integrable), multivalued mappings $T \to \mathrm{Conv}(P)$. For $Q \in M(T; P)$, we define

$$\varphi(t, l \mid Q) = \min\{\langle l, w\rangle_U : w \in Q(t)\}, \quad l \in E$$

where E is the closed unit sphere in U with centre at zero (that is, $\varphi(t, \cdot \mid Q)$ is a lower support functional for the convex compact set $Q(t)$).

Note that for each $Q \in M(T; P)$ one has $\varphi(\cdot, \cdot \mid Q) \in B$, where B is the Banach space of Carathéodory functions $\varphi : T \times E \to R$ with the natural norm

$$\|\varphi\|_B = \int_T \|\varphi(t,\cdot)\|_{C(E)} dt$$

We now define a new metric σ in $M(T;P)$ by

$$\sigma(Q_1,Q_2) = \|\varphi(\cdot,\cdot|Q_1) - \varphi(\cdot,\cdot|Q_2)\|_B$$

It can be shown that if $\sigma(Q_n, Q) \to 0$, then also $\rho(\text{sel } Q_n, \text{sel } Q) \to 0$. Hence it follows that in order to solve the reconstruction problem we need only construct a family $(\Lambda_h)_{h>0}$ of non-predicting ρ-regularizing operators Λ_h: $\Xi \to M(T;P)$, that is

$$\sup\{\sigma(\Lambda_h\xi, Q(\cdot|y)) : \xi \in \Xi_h(y)\} \to 0, \quad h \to 0$$

Then $(D_h)_{h>0}$, $D_h\xi = \text{sel } \Lambda_h\xi$ will be a ρ-regularizing family of non-predicting operators.

Such operators Λ, in turn, may be sought in the class of finite-staged dynamical position algorithms with multi-valued piecewise-constant realizations of the controls (FDPAMs)

$$\Lambda = \left((\tau_i)_{i=0}^m; (Z_i^*)_{i=0}^{m-1}; (G_i^*)_{i=0}^{m-1} \right) \tag{2.1}$$

where Z_i^* is a mapping $T \times T \times V^* \times V^* \times \text{Conv}(P) \to V^*$ and G_i^* is a mapping $T \times V^* \times V^* \to \text{Conv}(P)$. A controlled process for the FDPAM (2.1) and a function $\xi \in \Xi$ is defined as a pair of functions on T

$$(z(\cdot), Q(\cdot)) = (z(\cdot|D,\xi), Q(\cdot|D,\xi)) \in (T \to V^*) \times (T \to \text{Conv}(P))$$

formed by the rule

$$z(t_0) = \xi(t_0), \quad z(t) = Z_i^*(t, \tau_i, z(\tau_i), \xi(\tau_i), Q_i(\cdot)), \quad \tau_i < t \leq \tau_{i+1}, \quad i = 0,\dots,m-1$$
$$Q(t) = Q_i(t) = G_i^*(\tau_i, z(\tau_i), \xi(\tau_i)), \quad \tau_i < t \leq \tau_{i+1}, \quad i = 0,\dots,m-1$$

The algorithm (2.1) and the corresponding operator $\Lambda : \Xi \ni \xi \to \Lambda\xi = Q(\cdot) \in M(T;P)$, as mentioned previously, will be identified. Clearly, any such operator has the unpredictability property. We will now construct suitable families of FDPAMs.

Condition 1. A function $\omega(\cdot,\cdot) : [0,\infty) \times [0,\infty) \to [0,\infty)$ exists such that $\omega(\delta, h) \to 0$ as $\delta, h \to 0$ and

$$\|A(t)y(t) + B(t, y(t))w + f(t) - A(s)(y(s) + \eta) - B(s, y(s) + \eta)w - f(s)\|_{V^*} \leq \omega(\delta, h)$$

for any $t, s \in T$, $|t - s| \leq \delta$, $y \in Y$, $\eta \in S_h$, $w \in P$.

Define a family of FDPAMs as follows:

$$\Lambda_h = \left((\tau_{hi})_{i=0}^{m(h)}; (Z_{hi}^*)_{i=0}^{m(h)-1}; (G_{hi}^*)_{i=0}^{m(h)-1} \right) \tag{2.2}$$

$$Z_{hi}^*(t,s,z,x,F) = x, \quad t,s \in T, \quad z,x \in V^*, \quad F \in \text{Conv}(P)$$
$$G_{hi}^*(t,z,x) =$$
$$= \left\{ w \in P : \left\| \frac{x-z}{\tau_{hi} - \tau_{hi-1}} + A(\tau_{hi-1})z - B(\tau_{hi-1}, z)w - f(\tau_{hi-1}) \right\|_{V^*} \leq \omega(\delta, h) + 2hC_V/\delta \right\}$$

for $t \in [\tau_{hi}, \tau_{hi+1})$, $z, x \in V^*$; otherwise, $G_{hi}^*(t, z, x) = P$.

Let $\delta(h)$ be the diameter of the partition of T, i.e.

$$\delta(h) = \max\{\tau_{hi+1} - \tau_{hi} \mid i = 0,\dots,m(h)-1\}$$

Theorem. Assume that condition 1 is satisfied and in addition $\delta(h) \to 0$, $h/\delta(h) \to 0$ as $h \to 0$. Then (2.2) is a σ-regularizing family of FDPAMs at every point $y \in Y$.

Proof. Let $y \in Y$. Fix an arbitrary sequence $\{h_k\}, h_k > 0, h_k \to 0$, and consider an arbitrary controllable process $(z_k(\cdot), Q_k(\cdot))$ corresponding to the FDPAM (2.2) with $\xi = \xi_k \in \Xi_h(y)$, $h = h_k$. It is required to show that $\sigma(Q_k(\cdot), Q(\cdot \mid y)) \to 0$ as $k \to \infty$.

Suppose that this convergence fails to hold. Then, by the definition of σ

$$\int_T \|\varphi(t, \cdot \mid Q_k) - \varphi(t, \cdot \mid Q)\|_{C(E)} dt \not\to 0$$

Fix an arbitrary $\varepsilon > 0$ and choose some finite ε-net $\{l_1, \ldots, l_n\}$ for E. Then

$$\sup_{l \in E} \inf_{1 \le j \le n} \left\| l - l_j \right\|_U \le \varepsilon \tag{2.3}$$

Without loss of generality, we may assume, extracting a subsequence if necessary, that for every $j \in \{1, \ldots, n\}$

$$\varphi(\cdot, l_j \mid Q_k) \to \varphi^*(\cdot, l_j) \text{ weakly in } L^2(T; R)$$

where $\varphi^*(\cdot, l_j)$ is a function in $L^2(T; R)$. By the properties of lower support functions, it can be shown that for every $j \in \{1, \ldots, n\}$

$$\varphi^*(t, l_j) \ge \varphi(t, l_j \mid Q) \text{ for almost every } t \in T$$

It follows from the definition of a controlled process that, for any $u \in \Sigma(y)$ and any subscript k

$$[u]_i \in Q_k(\tau_{hi}), \quad i = 1, \ldots, m(h) - 1, \quad h = h_k$$

where $[\psi]_i$ is the value of the integral of ψ over the interval τ_{hi-1}, τ_{hi} |, $h = h_k$, divided by the length of the interval.

The above inclusion relations imply the inequalities

$$\varphi\left(\tau_{hi}, l_j \middle| Q_k\right) \le \left[\left\langle u(\cdot), l_j \right\rangle_U\right]_i, \quad h = h_k$$

A unique function $v_j \in \Sigma(y)$ exists such that

$$\langle v_j(\tau), l_j \rangle_U = \varphi(\tau, l_j \mid Q) \text{ for almost every } \tau \in \Gamma$$

For $u = v_j$ we get

$$\varphi\left(\tau_{hi}, l_j \middle| Q_k\right) \le \left[\varphi\left(\cdot, l_j \middle| Q\right)\right]_i, \quad \varphi\left(\tau_{hi}, l_j \middle| Q_k\right) \le \left[\varphi^*\left(\cdot, l_j\right)\right]_i, \quad h = h_k$$

Weak convergence and the inequalities just proved imply that

$$\varphi(\cdot, l_j \mid Q_k) \to \varphi^*(\cdot, l_j) \text{ strongly in } L^2(T; R) \tag{2.4}$$

We have $\varphi^*(\cdot, l_j) = \varphi(\cdot, l_j \mid Q)$.

Indeed, otherwise it would follow that (all integrations are over the interval Γ)

$$\int \varphi\left(t, l_j \middle| Q\right) dt < \int \varphi^*(t, l_j) dt$$

On the other hand

$$\int \varphi\left(t, l_j \middle| Q_k\right) dt \le \int \varphi\left(t, l_j \middle| Q\right) dt, \quad \int \varphi\left(t, l_j \middle| Q_k\right) dt \to \int \varphi^*\left(t, l_j\right) dt$$

Therefore

$$\int \varphi\left(t, l_j \middle| Q\right) dt = \int \varphi^*\left(t, l_j\right) dt$$

contrary to our assumption.

Using (2.3), we obtain the inequality

$$\left\|\varphi(t,\cdot|Q_k)-\varphi(t,\cdot|Q)\right\|_{C(E)}\leq \max_{1\leq j\leq n}\left|\varphi\left(t,l_j\middle|Q_k\right)-\varphi\left(t,l_j\middle|Q\right)\right|+\varepsilon\cdot L$$

where L is some positive constant independent of k, t and j. By (2.4) and the fact that ε is arbitrary, we have

$$\int\left\|\varphi(t,\cdot|Q)-\varphi(t,\cdot|Q_k)\right\|_{C(E)}dt\to 0$$

contrary to our initial assumption. Thus, $\sigma(Q_k(\cdot), Q(\cdot|y))\to 0$ as $k\to\infty$. This proves the theorem.

Remarks. 1. Concerning classes of partial differential equations that can be represented as system (1.1) in the functional-analytic formulation see, e.g. [14, 15].

2. Condition 1 is satisfied for problems in which the family of operators $\{A\{t\}: t\in T\}$ satisfies the additional condition of equicontinuity in some neighbourhood of the set of all states of system (1.1).

3. If U is an infinite-dimensional separable Hilbert space, then U can be equipped with a new scalar product, relative to whose norm the sphere E is compact [16]. Then the scheme of the above arguments (with the new scalar product and norm on U) can be extended without change to this infinite-dimensional case.

4. In practice, if the entire set of controls generating an observable motion has been determined, it is frequently convenient to take the Chebyshev centre of the set or an element of minimum norm as the solution of the reconstruction problem. It can be shown that the conditions of the problem considered above, converge of the sets in the Hausdorff metric implies convergence of their Chebyshev centres and elements of minimum norm [17].

5. A different method for the dynamical reconstruction of the set of controls generating an observable motion was described in [3].

This research was supported by the Russian Foundation for Basic Research (96-01-00846) and the International Centre of Science and Technology (94-008).

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Translated by D.L.